Observation of a high-energy tail in ion energy distribution in the cylindrical Hall thruster plasma
Youbong Lim, Holak Kim, Wonho Choe, Seung Hun Lee, Jongho Seon, and Hae June Lee

Citation: Physics of Plasmas (1994-present) 21, 103502 (2014); doi: 10.1063/1.4897178
View online: http://dx.doi.org/10.1063/1.4897178
View Table of Contents: http://scitation.aip.org/content/aip/journal/pop/21/10?ver=pdfcov
Published by the AIP Publishing

Articles you may be interested in
Internal plasma potential measurements of a Hall thruster using plasma lens focusing
Phys. Plasmas 13, 103504 (2006); 10.1063/1.2358331

Internal plasma potential measurements of a Hall thruster using xenon and krypton propellant
Phys. Plasmas 13, 093502 (2006); 10.1063/1.2335820

Physical characterization of high-frequency instabilities in Hall thrusters
Phys. Plasmas 13, 083503 (2006); 10.1063/1.2231723

Magnetically filtered Faraday probe for measuring the ion current density profile of a Hall thruster

Spectral analysis of Hall-effect thruster plasma oscillations based on the empirical mode decomposition
Phys. Plasmas 12, 123506 (2005); 10.1063/1.2145020
Observation of a high-energy tail in ion energy distribution in the cylindrical Hall thruster plasma

Youbong Lim, Holak Kim, Wonho Choe, Seung Hun Lee, Jongho Seon, and Hae June Lee

1Department of Physics, Korea Advanced Institute of Science and Technology, 291 Daehak-ro, Yuseong-gu, Daejeon 305-701, South Korea
2School of Space Research, Kyung Hee University, 1732 Deokyoungdaero, Gyeong-gu, Yongin-si, Gyeonggi-do 446-701, South Korea
3Department of Electrical Engineering, Pusan National University, Busandaehak-ro 63beon-gil, Geumjeong-gu, Busan 609-735, South Korea

(Received 3 July 2014; accepted 17 September 2014; published online 3 October 2014)

A novel method is presented to determine populations and ion energy distribution functions (IEDFs) of individual ion species having different charge states in an ion beam from the measured spectrum of an E × B probe. The inversion of the problem is performed by adopting the iterative Tikhonov regularization method with the characteristic matrices obtained from the calculated ion trajectories. In a cylindrical Hall thruster plasma, an excellent agreement is observed between the IEDFs by an E × B probe and those by a retarding potential analyzer. The existence of a high-energy tail in the IEDF is found to be mainly due to singly charged Xe ions, and is interpreted in terms of non-linear ion acceleration.

[http://dx.doi.org/10.1063/1.4897178]

I. INTRODUCTION

Ion sources have been developed for various scientific and technological applications such as electric propulsion for spacecraft, ion implantation for material modification, and neutral beam injection for nuclear fusion devices. Among many physical parameters characterizing the produced ion beam, the ion energy distribution function (IEDF) and the fraction of multiply charged ions are important parameters that should be quantized. In particular, the fraction of ions with different charges directly affects propellant utilization and overall efficiency in an electric propulsion system. A Hall thruster is a spacecraft propulsion device having crossed electric and magnetic fields. A cylindrical type Hall thruster has been designed especially for low power operations by using crossed electric and magnetic fields. The peaks of multiply charged ions. These characteristics of the probe spectrum originate from the presence of a finite range of ionization and acceleration zones in the Hall thruster plasma, and from the finite velocity resolution of the E × B probe. Several fitting methods using triangle, Gaussian, and exponential functions have been developed to determine the fractions of ions having different charge states in Hall thruster plasmas. Since all these methods basically assume the constant velocity resolution of the E × B probe, the population of each ion obtained from a simple calculation of the area under each corresponding current peak has limited accuracy because the velocity resolution of the E × B probe depends on the energy and charge state of the beam ions. Therefore, the velocity axis, on which an E × B probe spectrum is defined, should be properly transformed by considering the complicated velocity resolution, and we provide novel mathematical method to transform an E × B probe spectrum into IEDFs of ions having different charge states. Although it has already been proposed in a somewhat less accurate manner in the work by Renaud et al., our approach is more complete as it treats the velocity distribution, not just one single velocity component.

This paper is organized as follows. In Sec. II, we provide the experimental setup for the measurements of E × B probe spectra. The reconstruction method to obtain IEDFs from the measured spectra is presented in Sec. III. In Sec. IV, we analyze the IEDFs reconstructed for Hall thruster plasmas, and the high-energy tail formation in the IEDFs is investigated in terms of stochastic ion acceleration. Section V summarizes our major findings.

II. EXPERIMENTAL SETUP

An E × B probe is a velocity filter that selects ions according to their velocities by using crossed electric and magnetic fields.
magnetic fields. Most probes normally establish a constant magnetic field by using permanent magnets, while the electric field is variable by sweeping plate potential.\(^1\)

As illustrated in Fig. 1, the Xe ion beam was produced by a cylindrical type Hall thruster whose channel length and outer channel diameter are 25 and 50 mm, respectively. The details of the thruster used in the experiment are described in Refs. 3 and 4. Through the experiment, the Xe flow rates to the anode and cathode were fixed at 7.0 and 0.7 sccm, respectively, and the anode voltage was set at 225 V. Under this condition, the discharge current was measured to be 1.32 A. The experiments were performed in a 1.5 m in diameter and 3 m long vacuum chamber on which two cryopumps (U22H, Ulvac) are installed. The operating pressure of about \(6.0 \times 10^{-6}\) Torr for Xe is typically obtained at a continuous Xe flow of 7.7 sccm.

As illustrated in Fig. 2(a), the main components of the \(E \times B\) probe under experiment are an entrance collimator, a velocity filter, an exit collimator, and a collector. The dimension of both entrance and exit collimators is 70 mm in length and 4 mm in orifice diameter, and the collimators are made of stainless steel. The length of the velocity filter is 140 mm, in which perpendicular electric and magnetic fields are applied. The magnetic field in the velocity filter section is provided by permanent magnets, and the magnetic flux is guided by iron plates having a thickness of 4 mm to provide homogeneous magnetic field inside the velocity filter. The field strength was measured to be 0.23 T at the center of the velocity filter. The electric field is applied by a pair of aluminum plates separated by 10 mm from each other. The entrance collimator is positioned at 600 mm away from the thruster exit plane. The ion beam current is collected by a collector made of stainless steel and measured by a picoameter (KEITHLEY 6485). The probe body is electrically grounded and it is evacuated through several holes in the body to limit collisions through the drift section.

A RPA was also installed to measure IEDF by using a series of grids to selectively filter ions. The RPA used for the experiment [Fig. 2(b)] has an entrance collimator identical to the one used in the \(E \times B\) probe. Detailed information on the RPA is found in Ref. 7 and the references therein. The collimated RPA was mounted on a rotating stage centered at and 500 mm away from the thruster exit.

III. IEDF RECONSTRUCTION METHOD

An \(E \times B\) probe selects ions having a specific speed that satisfy the Lorentz force balance condition under externally provided electric and magnetic fields. The measurable of the diagnostic is the selected ion beam current given as a function of applied plate voltage, i.e., electric field strength in the velocity filter. In addition, this study aimed to obtain the accurate IEDF of each charge state ion based on the measured ion current. If it is supposed that \(g\) is the measured \(E \times B\) probe spectrum as a function of plate voltage and \(f\) is the corresponding energy distribution function of the ion beam, \(f\) and \(g\) represented by column vectors (an \(M \times 1\) and an \(N \times 1\) matrix, respectively) are related with each other via a characteristic matrix \(A\) (an \(N \times M\) matrix) as

\[ Af = g. \] (1)

In order to obtain \(f\) from Eq. (1), \(A\) should be specified for each charge state ion, i.e., \(Xe^+, Xe^{2+}, Xe^{3+}\), etc.
respectively. The characteristic matrix $A$ is set up by simulating the $E \times B$ probe spectrum $G_{W_{i}}$ for mono-energetic ion beams whose kinetic energies are $W_{i}$

$$G_{W_{i}} = [G_{W_{1}(V_{1})}, G_{W_{2}(V_{2})}, \cdots, G_{W_{N}(V_{N})}]^{T},$$

where $V_{i}$ is the plate voltage of the $E \times B$ probe, and $M$ and $N$ are the number of elements on the axis of the ion energy and of the plate voltage, respectively. The probe spectrum $g$ for an ion beam having a specific IEDF $f$ is obtained by a linear superposition of $G_{W_{i}}$, as

$$f = [f(W_{1}), f(W_{2}), \cdots, f(W_{N})]^{T},$$

$$g = f(W_{1}) G_{W_{1}} + f(W_{2}) G_{W_{2}} + \cdots + f(W_{M}) G_{W_{M}}.$$

Based on Eqs. (1)–(4), $A$ is expressed in terms of the calculated spectra as

$$A = [G_{W_{1}} G_{W_{2}} \cdots G_{W_{M}}].$$

The probe spectra $G_{W_{i}}$ for mono-energetic ion beams are constructed based on the calculated trajectories of the ion beam. One thousand test ions were used to represent a mono-energetic ion beam and the initial velocity of each ion was given randomly within the range determined by the geometry of the entrance collimator. Figure 3 shows the ion trajectories calculated by solving the equation of motion under a magnetic field and an electric field. The topologies of both fields were calculated by using the finite element method magnetics (FEMM) code based on the experimental condition. The magnetic field provided by permanent magnets was calibrated by measuring the field strength along the probe’s centerline using a Gauss meter. The electric field was calculated under different plate voltages from $V_{1}$ to $V_{N}$ and then different ion energies from $W_{1}$ to $W_{M}$ to determine $A$ of Eq. (5). The characteristic matrix $A$ contains all the information such as the dimensions of the $E \times B$ probe, the electric and magnetic fields inside the $E \times B$ probe, and mass and charge state of the beam ions.

Shown in Fig. 4 are the calculated $A$ for $\text{Xe}^{+}$, $\text{Xe}^{2+}$, and $\text{Xe}^{3+}$ ions, respectively, where the abscissa and the ordinate are the ion beam energy and the plate voltage. It is shown that the broadening in the distributions of the row vectors of $A$ is linearly increased with respect to the plate voltage. That means that the ions within an energy band can reach the collector at a given plate voltage, which represents the velocity resolution of the $E \times B$ probe. This linear broadening of the probe resolution was also pointed out in an analytic manner by Kim.10

It is important to emphasize a trade-off between the energy resolution and signal-to-noise ratio of an $E \times B$ probe. A high-energy resolution of the probe is obtainable simply with small orifice and long collimation length of the probe, and the resulting signal is drastically decreased by the small resolution. If one can measure probe signal with infinitesimal probe resolution, the plate voltage can be directly converted into the velocity and energy of ions. Figure 4 demonstrates that the velocity resolution of the $E \times B$ probe should be considered, particularly for ions having higher energy and lower charge state. It was also pointed out that...
the resolution is not negligible in analyzing \( E \times B \) probe spectra by Hofer.\(^{11}\)

In order to reconstruct the IEDF \( f \) from the measured \( E \times B \) probe spectrum \( g \), it is necessary to obtain an inverse matrix \( A^{-1} \) that is represented by an \( M \times N \) matrix

\[
f = A^{-1} g.
\]

(6)

Since \( A \) tends to be numerically singular due to the finite velocity resolution of the \( E \times B \) probe, the inverse matrix cannot be directly obtained. In this work, we adopted the Tikhonov regularization method to determine the inverse transformation.\(^{12,13}\)

The linear system defined by Eq. (1) is proposed to find a particular solution for \( f \), which minimizes the following residual:

\[
R = \| Af - g \|^2.
\]

(7)

This system is under-determined due to the singularity of \( A \), and the regularization term is, therefore, included in the original residual as follows:

\[
R = \| Af - g \|^2 + k \| Tf \|^2,
\]

(8)

where \( T \) is the Tikhonov matrix that is chosen as the Laplacian matrix to obtain a smooth solution, and \( k \) is the relative importance of the smoothness. The regularized solution to minimize the residual of Eq. (8) is given by

\[
f_r = \left( A^T A + k T^T T \right)^{-1} A^T g.
\]

(9)

As an example, the blue curve shown in Fig. 3(a) demonstrates the IEDF \( f \) from a Gaussian spectrum \( g \), obtained by Eq. (9). As depicted in the figure, \( f \) fluctuates at the ion energy over 300 eV, and it becomes negative, which is unacceptable. Although the fluctuation level can be mitigated by increasing the regularization strength \( k \), the resulting solution becomes inaccurate due to the decreased fraction of the original residual of Eq. (7) in the regularized residual of Eq. (8). Therefore, an accurate and physically acceptable solution cannot be obtained by using this conventional regularization method.

This problem is avoided by an iterative Tikhonov regularization method. It is designed to minimize the residual including the similarity term as\(^{12,13}\)

\[
R_s = \| Af - g \|^2 + k_s \| f - f_0 \|^2,
\]

(10)

where \( f_0 \) is the assumed solution and \( k_s \) is the relative importance of similarity with \( f_0 \). The solution for \( f \) to minimize \( R_s \) is obtained by

\[
f_r = \left( A^T A + k_s I \right)^{-1} (A^T g + k_s f_0),
\]

(11)

where \( I \) is an identity matrix. Now, Eq. (11) is iterated with \( f_0 \) where

\[
f_0 = \| f_r \|,
\]

(12)

and \( f_r \) is the obtained solution at the previous step. The assumed solution for the first iteration is obtained from the conventional method of Eq. (9). The solution of the iterative regularization method, therefore, can be written in a simple recurrence relation as

\[
f_{r+1} = \left( A^T A + k_s I \right)^{-1} (A^T g + k_s |f_r|),
\]

(13)

where \( i \) is the iteration number. The red curve depicted in Fig. 5(a) is the IEDF reconstructed in this way from the same \( g \) with 100 iterations, which shows no fluctuation and negative value. The reconstruction error \( R_i \) in Fig. 5(b) is defined by the difference in the original residual with the reconstructed solution \( f_i \) as

\[
R_i = \| A f_i - g \|^2.
\]

(14)

As shown in the figure, only 100 iterations show a \( 10^{-4} \) times reduction in the reconstruction error compared with the non-iterated solution. This iterative reconstruction method, therefore, finds accurate IEDFs from the measured \( E \times B \) probe spectra.

IV. RESULTS AND DISCUSSIONS

Based on the developed method, IEDFs were reconstructed from the measured \( E \times B \) probe spectrum obtained from the cylindrical type Hall thruster plasma. As shown in Fig. 6(a), three distinct peaks are observed in the measured probe spectrum, peaks corresponding to \( \text{Xe}^+ \), \( \text{Xe}^{2+} \), and \( \text{Xe}^{3+} \) ions, respectively. Since the characteristic matrix \( A \) is defined for each charge state ion, the peaks were separated by Gaussian fitting before starting the aforementioned reconstruction procedure. The individually reconstructed IEDFs for \( \text{Xe}^+ \), \( \text{Xe}^{2+} \), and \( \text{Xe}^{3+} \) ions are denoted as the solid curves in Fig. 6(b). Based on the conventional method that assumes a constant velocity resolution for an \( E \times B \) probe, the IEDFs are directly obtained from the separated peaks of the probe spectrum, which are plotted in the figure as the dashed curves for comparison. Each pair of the IEDFs for the same charge state is compared to show that the ion energy of all

![Graphs showing reconstructed IEDFs](image_url)
charge state ions and ion species fractions for the multiply charged ions are overestimated by the conventional method. In order to confirm the validity of our reconstruction method, the result was compared with the measured RPA result. It is noteworthy that the IEDF measured by the RPA denoted as black dots in the figure. Each of the IEDFs for $\text{Xe}^+$, $\text{Xe}^{2+}$, and $\text{Xe}^{3+}$ ions is shown as well.

It is important to emphasize that the overall IEDF shown in Fig. 6(c) is asymmetric with respect to the central energy (195 eV) with more high energy (>225 eV) fraction while the plasma is discharged at the anode voltage of 225 V. The reconstructed IEDFs show that the high-energy tail is mainly due to $\text{Xe}^+$ ions. Such high-energy tail may be explained by a wave-riding mechanism, i.e., non-linear ion acceleration along the oscillating electric field.\textsuperscript{14–18} As a validation of our argument, we performed numerical calculations for a test ion in an oscillating plasma potential whose Hamiltonian takes the form,

$$H = \frac{P_x^2}{2m} + e V_p(x,t),$$  \hspace{1cm} (15)

where $P_x$ is the canonical momentum such that $P_x = mv_x$, and $V_p$ is the oscillating plasma potential. When the effective residence time of the ion inside the discharge channel is compared with the oscillation frequency of the electric field, the test ion can gain energy beyond the anode voltage. The numerical calculations verify that the maximum ion energy increases as the oscillation amplitude increases. In addition, the maximum energy of the singly charged ions can be much higher than that of the multiply charged ions, which agrees with the reconstructed IEDFs. Therefore, the IEDFs of Hall thruster plasmas are significantly affected by fluctuation characteristics. Further investigations will be performed to understand the underlying physics for the high energy ions, and the results will be reported elsewhere.

V. SUMMARY

We found that the velocity resolution of an $E \times B$ probe is not constant and depends on the kinetic energy and charge state of the ions. The measured $E \times B$ probe spectrum ($g$) is related with the energy distribution of the source ions ($f$) via $\mathbf{A} \mathbf{f} = \mathbf{g}$, and the aim of this paper is to find $\mathbf{f}$ from the measured $\mathbf{g}$, which is an inverse problem. The characteristic matrix $\mathbf{A}$ was calculated by simulating $\mathbf{g}$ due to the monoenergetic ion beams with different energies, and the inversion was performed based on the iterative Tikhonov regularization. Using this developed method, we found excellent agreement between the IEDFs by the $E \times B$ probe and the RPA, and the existence of high-energy ions in the cylindrical type Hall thruster plasma owing to higher energy $\text{Xe}^+$ ions.

ACKNOWLEDGMENTS

This work was partly supported by the Space Core Technology Program (Grant No. 2014M1A3A3A02034510).
through the National Research Foundation of Korea funded by Ministry of Science, ICT and Future Planning, and also partly supported by the Korea Institute of Materials Science (KIMS) (Grant No. 10043470) funded by the Ministry of Trade, Industry and Energy of Korea.

10S. W. Kim, Ph.D. dissertation, Department of Aerospace Engineering, University of Michigan, Ann Arbor, MI, 1998.